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Spectral function sum rules and higher-mass vector mesons

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Abstract. Contributions of the ρ' meson are computed to various spectral function sum rules. We conclude the possible existence of higher-mass axial vector mesons, isoscalar vector mesons and the importance of the higher-mass continuum, respectively.

1. Introduction

Recently, it has been shown (Barbarino *et al* 1972, Borgia *et al* 1972) that the cross section of the process $e^+e^- \rightarrow 2\pi^+2\pi^-$ forms a broad peak around $2E \simeq 1{\cdot}6$ GeV which can be explained by assuming a new vector meson, ρ', with the quantum numbers of ρ and $m_{\rho'} = 1{\cdot}6$ GeV, $\Gamma_{\rho'} = 350$ MeV (Borgia *et al* 1972, Bramon and Greco 1972a). For the strength $em_{\rho'}^2/f_{\rho'}$ of the γ–ρ' coupling several proposals have been made, presumably $10 \leqslant f_{\rho'}^2/4\pi \leqslant 18$ (Borgia *et al* 1972, Bramon and Greco 1972a, Ceradini *et al* 1972).

The existence of the ρ' state is shown also by the analysis of the photoproduction $\gamma p \rightarrow p2\pi^+2\pi^-$, whence $m_{\rho'} = 1{\cdot}43 \pm 0{\cdot}05$ GeV, $\Gamma_{\rho'} = 0{\cdot}65 \pm 0{\cdot}1$ GeV, $f_{\rho'}^2/4\pi = 15{\cdot}6$ (Smadja *et al* 1972, Bingham *et al* 1972a, b).

Beside the experimental activity, there are many theoretical attempts for introducing higher-mass vector states including Regge-type models (Shapiro 1969, Veneziano 1968), various descriptions of the extended vector meson dominance (Bramon and Greco 1972b, Bramon *et al* 1972, Sakurai and Schildknecht 1972a, b, c) and relativistic quark model calculations (Böhm *et al* 1972).

In the present note we show that the spectral function sum rules suggest the presence of higher-mass vector states, too. Actually, the saturation of the spectral function sum rules by known resonances and ρ' claims further contributions of unknown axial vector and isoscalar vector states within the error bars of the input data.

2. Axial vector sum rule

Consider first the original Weinberg sum rules (Weinberg 1967, for an implicit form see Pócsik 1966) in π, ρ, A_1, ρ' approximation

$$\frac{m_\rho^2}{f_\rho^2}+\frac{m_{\rho'}^2}{f_{\rho'}^2} = \frac{m_A^2}{f_A^2}+c_\pi^2,$$
$$\frac{m_\rho^4}{f_\rho^2}+\frac{m_{\rho'}^4}{f_{\rho'}^2} = \frac{m_A^4}{f_A^2}. \qquad (1)$$

As is well known, for $f_{\rho'} \to \infty$

$$m_A^2 = 2m_\rho^2 \tag{2}$$

and

$$\frac{m_\rho^2}{2f_\rho^2} = c_\pi^2 \tag{3}$$

satisfy (1). Equations (2) and (3) hold well experimentally. For finite $f_{\rho'}$ substituting (2), (3) into (1), we get $m_{\rho'}/m_\rho = \sqrt{2}$ instead of the experimental ratio 2–2·1. It is easy to check that this failure of the ratio $m_{\rho'}/m_\rho$ is not a consequence of the small uncertainties of (2) and (3). Indeed, moving the input data π, ρ, A_1, ρ' (Rosenfeld *et al* 1973, Borgia *et al* 1972) within their errors bars, one can verify in the interval $10 \leqslant f_{\rho'}^2/4\pi \leqslant 20$ that the equation

$$\frac{m_\rho^2}{f_\rho^2}\left(1-\frac{m_\rho^2}{m_A^2}\right) - c_\pi^2 = \frac{m_{\rho'}^2}{f_{\rho'}^2}\left(\frac{m_{\rho'}^2}{m_A^2}-1\right) \tag{4}$$

following from (1) is considerably violated. For instance, in order to maintain (4), $f_{\rho'}^2/4\pi \simeq O(10^3)$ would be necessary (equivalently $m_A^2/m_\rho^2 \simeq$ 5–6). A small modification of the second Weiberg sum rule (Cook *et al* 1968) does not help.

Let us show that the Weinberg sum rules (1) cannot be improved by taking into account higher-mass contributions to the longitudinal spectral function of the axial current. Making use of Drell's (1972) method, represent the pseudoscalar continuum by a not necessarily observable heavy pion π' and write

$$\langle 0|A_{r\mu}(0)|\pi_t'(q)\rangle = \delta_{rt}\frac{iq_\mu c_\pi'}{\sqrt{(2Vq_0)}}. \tag{5}$$

π' changes (1), so that on the right hand side of (1) $c_\pi^2 \to c_\pi^2 + c_\pi'^2 = (X+1)c_\pi^2$. It is still not excluded that the non-pion continuum is $O(m_\pi^2)$, that is $X \lesssim 0.01$ (Furlan *et al* 1972). In this case (1) is still wrong. Let us look at the case of the weak PCAC (Drell 1972) with $X \simeq 1$. Then, we get from (1), (2), (3) exactly $m_{\rho'}/m_\rho \leqslant \sqrt{2}$ and increasing X yields decreasing $m_{\rho'}/m_\rho$. Similarly, the analogue of (4) results in $f_{\rho'}^2 < 0$ for $0.01 \lesssim X$ within the error bars of the input data. In general, increasing corrections to PCAC make the agreement worse.

We conclude that the transversal spectral function of the axial current is responsible for the failure of (1) and it is also essential to take into account the higher-mass axial vector contributions. Therefore, let us assume there exists a daughter of A_1 named A_1' which is the chiral $SU_2 \times SU_2$ partner of ρ' restoring the balance in (1). Strictly speaking, we do not know whether A_1' is a real axial vector meson, however, it represents at least the higher continuum in (1) (as π' in PCAC). In this case (1) merely restricts the unknown parameters f_A, $f_{A'}$, $m_{A'}$ in terms of the others where

$$\langle 0|A_{r\mu}(0)|A_{1t}'(k)\rangle = \frac{\mathscr{E}_\mu}{\sqrt{(2Vk_0)}}\frac{m_{A'}^2}{f_{A'}}\delta_{rt}. \tag{6}$$

As a real resonance, A_1' participates in the same processes as A_1 and in the intermediate state $A_1'\pi$ it contributes to the total e^+e^- annihilation cross section into positive G-parity final states. In order to estimate its mass, we assume that the A_1 vertex does not depend too strongly on ρ', A_1', c_π', that is the low lying π, ρ, A_1 satisfy the Weinberg sum rules, then

$$\frac{m_{\rho'}^2}{f_{\rho'}^2} = \frac{m_{A'}^2}{f_{A'}^2} + c'^2_\pi$$

$$\frac{m_{\rho'}^4}{f_{\rho'}^2} = \frac{m_{A'}^4}{f_{A'}^2} \tag{7}$$

whence

$$m_{A'}^2 = m_{\rho'}^2\left[1-\left(\frac{f_{\rho'}c_\pi'}{m_{\rho'}}\right)^2\right]^{-1}. \tag{8}$$

(8) gives $m_{A'} \simeq 1.6\,\text{GeV}$ for $X \lesssim 0{\cdot}01$ and $2{\cdot}1\,\text{GeV} \leqslant m_{A'} \leqslant 5{\cdot}1\,\text{GeV}$ for $10 \leqslant f_{\rho'}^2/4\pi \leqslant 20$ with large corrections to PCAC, $X \simeq 1$. In this case the well known KSFR combination (see (3))

$$\frac{f_{\rho'}^2 c_\pi^2}{m_{\rho'}^2} = \frac{1}{2a} \tag{9}$$

determines the mass $m_{A'}$.

In connection with (9) we discuss the validity of the KSFR relation for ρ', that is $a = 1$. From experimental data (9) gives $0{\cdot}6 \leqslant a \leqslant 1{\cdot}2$ for $20 \geqslant f_{\rho'}^2/4\pi \geqslant 10$. Denoting the decaying $\rho'\pi\pi$ form factor by $F(p_{\rho'}^2, k_\pi^2, q_\pi^2)$, we get from current algebra and PCAC for narrow ρ' (Kawarabayashi and Suzuki 1966)

$$F(0,0,0) = \frac{1}{2c_\pi^2}\frac{m_{\rho'}^2}{f_{\rho'}}, \tag{10}$$

thus $F(0,0,0) = af_{\rho'}$. This relatively high value of $F(0,0,0)$ shows that a large extrapolation $F(0,0,0) \rightarrow F(m_{\rho'}^2, m_\pi^2, m_\pi^2)$ is responsible for the small $\rho'\pi\pi$ decay.

3. Further sum rules

In the octet picture of currents we have (Das *et al* 1967) from the first Weinberg sum rule with $i = 3, j = 8$ in ρ, ω, ϕ, ρ' approximation

$$\sum_{i=\omega,\phi} m_i\Gamma(i\rightarrow l\bar{l}) = \tfrac{1}{3}\sum_{i=\rho,\rho'} m_i\Gamma(i\rightarrow l\bar{l}). \tag{11}$$

Numerical results are indicated in table 1. Combining the contributions (or different rows) in table 1, we conclude that within the error bars of the input data the agreement with (11) is better for larger values of $f_{\rho'}$ (with decreasing ρ' contributions), however, the existence of further ω-type vector states is still necessary. The corresponding second Weinberg sum rule saturated by ρ, ω, ϕ, ρ' is badly broken (table 1), but the presence of the large ρ' contributions requires again new ω-type states.

Consider now the situation of the improved second Weinberg sum rule (Das *et al* 1967), including also ρ' one writes

$$-\tfrac{1}{3}\sum_{i=\rho,\rho'} m_i\Gamma(i\rightarrow l\bar{l})(m_i^2-4m_{K^*}^2) = 3\sum_{i=\omega,\phi} m_i^3\Gamma(i\rightarrow l\bar{l}) \tag{12}$$

Table 1

Reference of input data	Contribution from $\omega+\phi$	Contribution from ρ	$\frac{f_{\rho'}^2}{4\pi}$	Contribution from ρ'	
Rosenfeld *et al* (1973)	2	1·4	10	1·5	equation (11) in MeV^2
Benaksas *et al* (1972) and Lefrancois (1971)	2·2	1·6	17	0·9	
Rosenfeld *et al* (1973)	1·9	0·8	10	3·9	the second Weinberg sum rule in MeV^2 GeV^2
Benaksas *et al* (1972) and Lefrancois (1971)	2	0·9	17	2·3	
Rosenfeld *et al* (1973)	5·6	3·7	10	0·9	equation (12) in in MeV^2 GeV^2
Benaksas *et al* (1972) and Lefrancois (1971)	6·1	4	17	0·5	

where we have used the $i = 3, j = 4$ first Weinberg sum rule in ρ, ρ', K^* approximation. The agreement becomes in general better for larger ρ' contributions (lower $f_{\rho'}$; see table 1) within the error bars of the input data which new ρ' states could improve.

Finally, (11) and (12), as exact relations, give a negative answer for $\Gamma(\phi \rightarrow ll)$.

In summary, the large ρ' contributions in the spectral function sum rules involve the existence of new significant vector and axial vector states. The discrete resonance or continuum character of these states will be shown by high-energy (among others e^+e^-) experiments.

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